Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1802389966

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2020**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

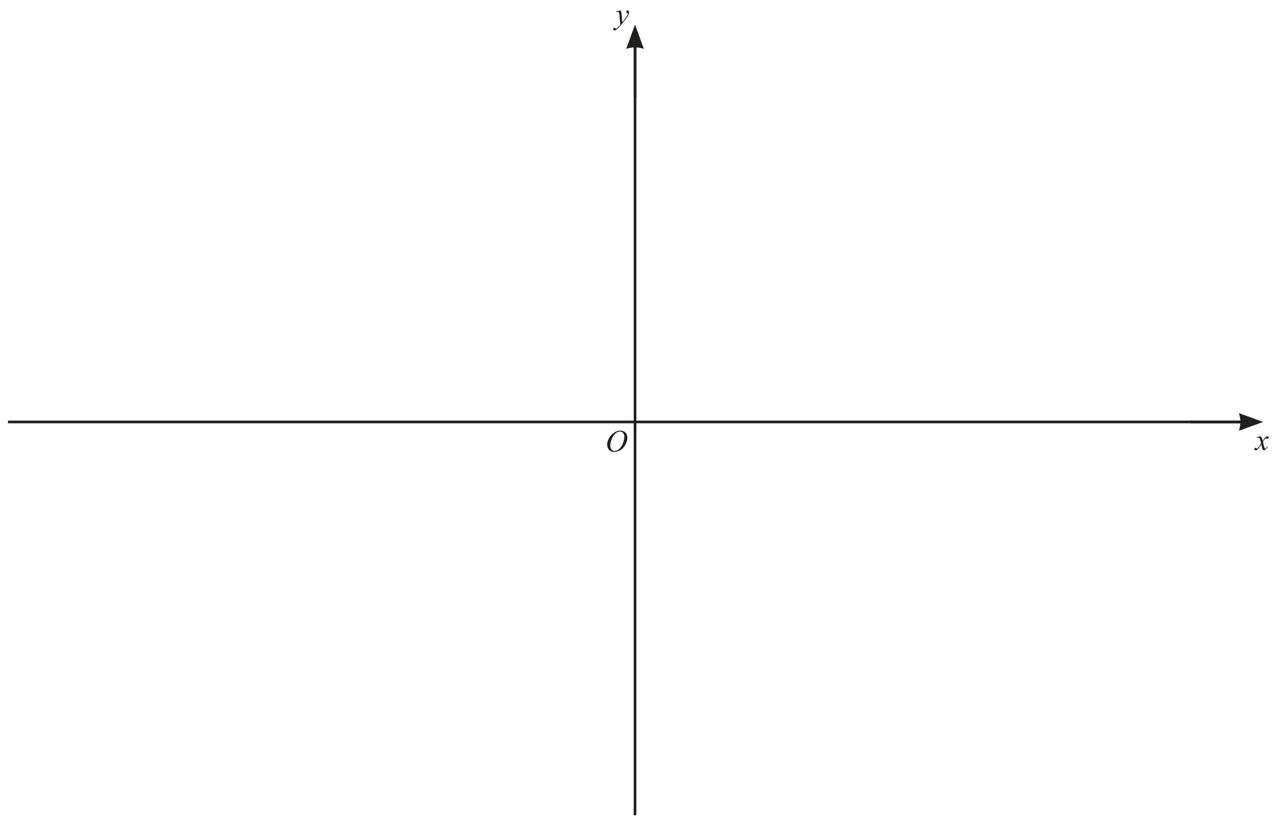
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 On the axes below, sketch the graph of $y = |(x-2)(x+1)(x+2)|$ showing the coordinates of the points where the curve meets the axes. [3]



2 The volume, V, of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

The radius, r cm, of a sphere is increasing at the rate of $0.5\,\text{cm s}^{-1}$. Find, in terms of π, the rate of change of the volume of the sphere when $r = 0.25$. [4]

3 **(a)** Find the first 3 terms in the expansion of $\left(4-\frac{x}{16}\right)^6$ in ascending powers of x. Give each term in its simplest form. [3]

(b) Hence find the term independent of x in the expansion of $\left(4-\frac{x}{16}\right)^6\left(x-\frac{1}{x}\right)^2$. [3]

4 **(a)** **(i)** Find how many different 5-digit numbers can be formed using the digits 1, 2, 3, 5, 7 and 8, if each digit may be used only once in any number. [1]

(ii) How many of the numbers found in **part (i)** are not divisible by 5? [1]

(iii) How many of the numbers found in **part (i)** are even and greater than 30 000? [4]

(b) The number of combinations of n items taken 3 at a time is 6 times the number of combinations of n items taken 2 at a time. Find the value of the constant n. [4]

5 $$\mathrm{f} : x \mapsto (2x+3)^2 \quad \text{for } x > 0$$

(a) Find the range of f. [1]

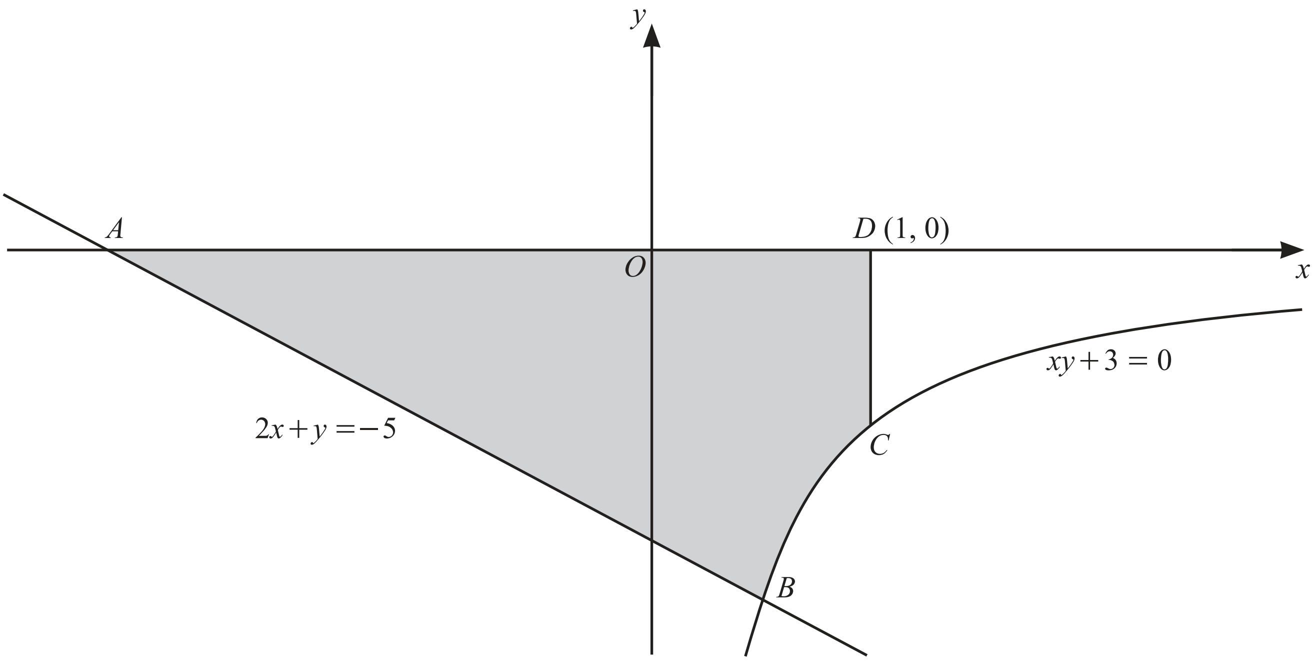
(b) Explain why f has an inverse. [1]

(c) Find f^{-1}. [3]

(d) State the domain of f^{-1}. [1]

(e) Given that $\mathrm{g} : x \mapsto \ln(x+4)$ for $x > 0$, find the exact solution of $\mathrm{fg}(x) = 49$. [3]

6



The diagram shows the straight line $2x + y = -5$ and part of the curve $xy + 3 = 0$. The straight line intersects the x-axis at the point A and intersects the curve at the point B. The point C lies on the curve. The point D has coordinates $(1, 0)$. The line CD is parallel to the y-axis.

(a) Find the coordinates of each of the points A and B. [3]

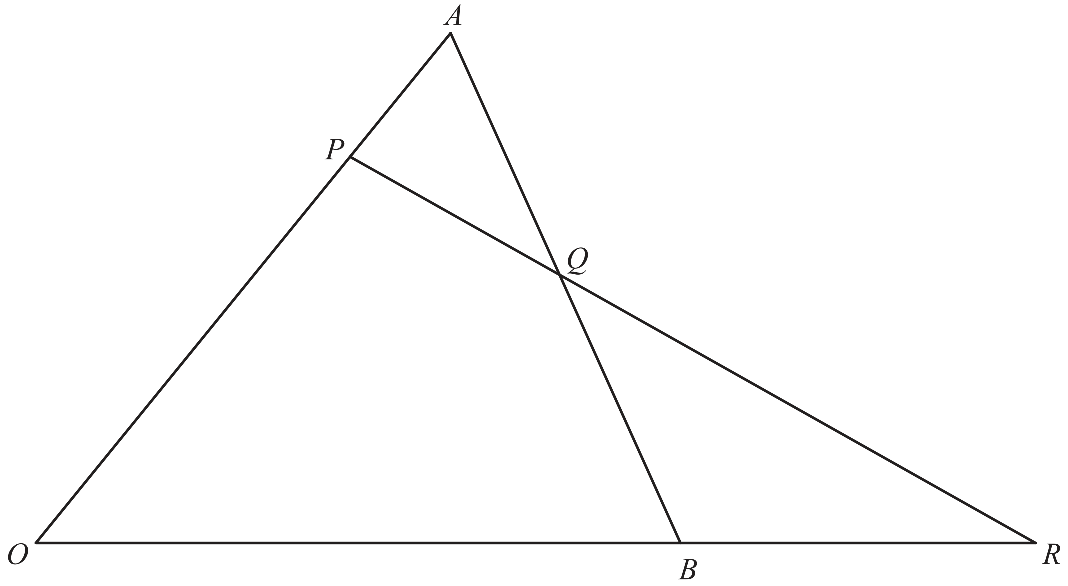
(b) Find the area of the shaded region, giving your answer in the form $p + \ln q$, where p and q are positive integers. [6]

7 **(a)** Given that $y=\left(x^2-1\right)\sqrt{5x+2}$, show that $\dfrac{dy}{dx}=\dfrac{Ax^2+Bx+C}{2\sqrt{5x+2}}$, where A, B and C are integers. [5]

(b) Find the coordinates of the stationary point of the curve $y = (x^2 - 1)\sqrt{5x+2}$, for $x > 0$. Give each coordinate correct to 2 significant figures. [3]

(c) Determine the nature of this stationary point. [2]

8



The diagram shows a triangle OAB such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P lies on OA such that $OP = \frac{3}{4}OA$. The point Q is the mid-point of AB. The lines OB and PQ are extended to meet at the point R. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(a) $\overrightarrow{AB}$, [1]

(b) $\overrightarrow{PQ}$. Give your answer in its simplest form. [3]

It is given that $n\overrightarrow{PQ} = \overrightarrow{QR}$ and $\overrightarrow{BR} = k\mathbf{b}$, where n and k are positive constants.

(c) Find $\overrightarrow{QR}$ in terms of n, **a** and **b**. [1]

(d) Find $\overrightarrow{QR}$ in terms of k, **a** and **b**. [2]

(e) Hence find the value of n and of k. [3]

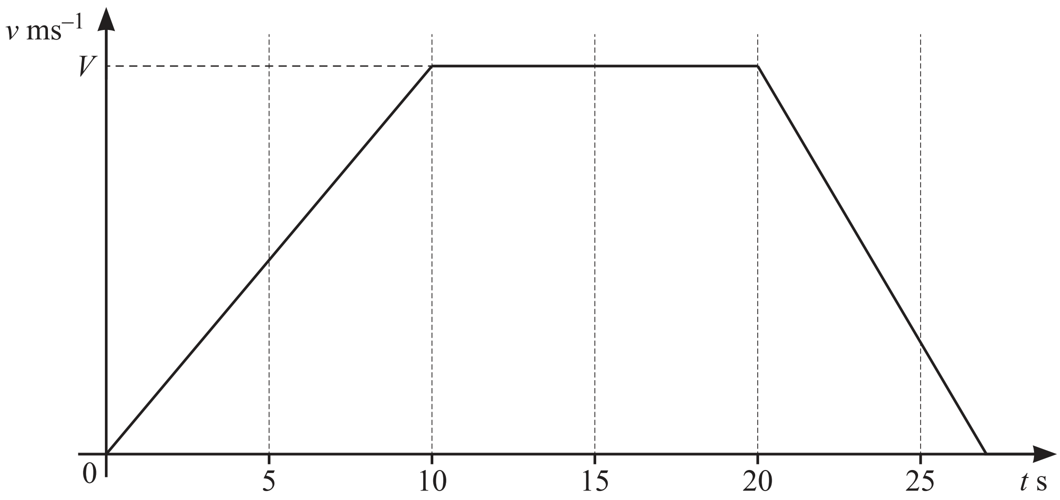
9 **(a)** A particle P moves in a straight line such that its displacement, x m, from a fixed point O at time t s is given by $x = 10\sin 2t - 5$.

(i) Find the speed of P when $t = \pi$. [1]

(ii) Find the value of t for which P is first at rest. [2]

(iii) Find the acceleration of P when it is first at rest. [2]

(b)



The diagram shows the velocity–time graph for a particle Q travelling in a straight line with velocity $v\,\text{ms}^{-1}$ at time $t\,\text{s}$. The particle accelerates at $3.5\,\text{ms}^{-2}$ for the first $10\,\text{s}$ of its motion and then travels at constant velocity, $V\,\text{ms}^{-1}$, for $10\,\text{s}$. The particle then decelerates at a constant rate and comes to rest. The distance travelled during the interval $20 \leqslant t \leqslant 25$ is $112.5\,\text{m}$.

(i) Find the value of V. [1]

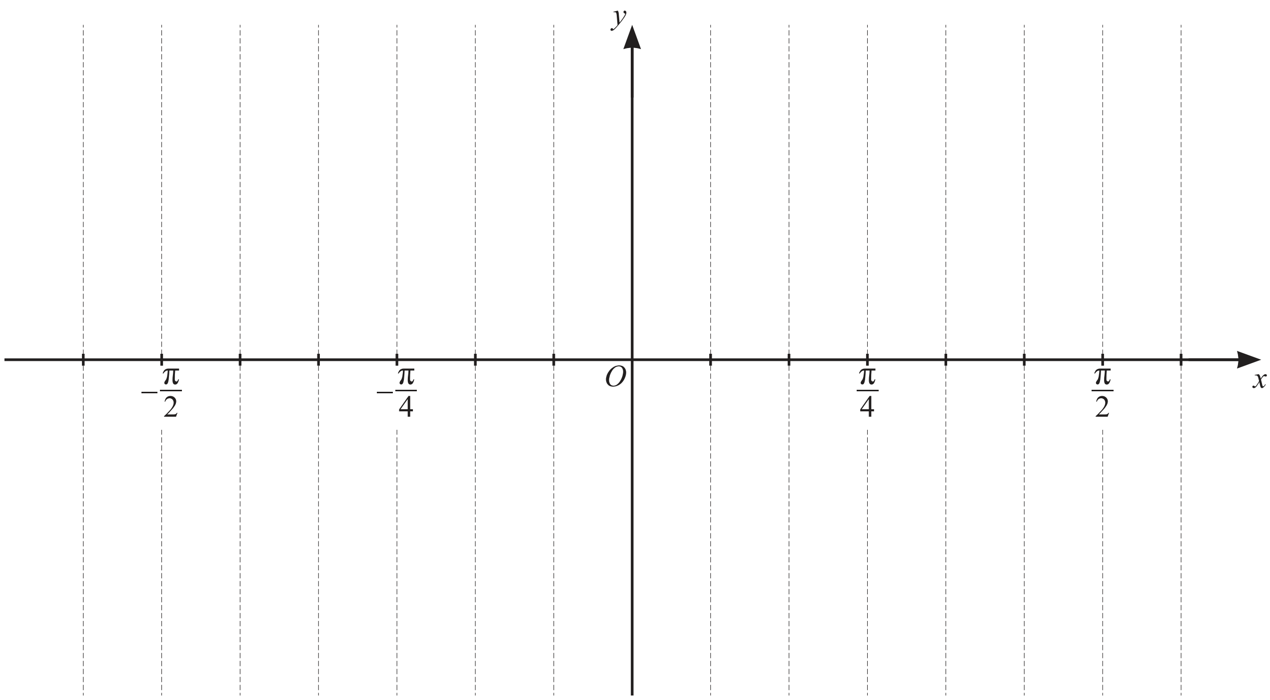
(ii) Find the velocity of Q when $t = 25$. [3]

(iii) Find the value of t when Q comes to rest. [3]

Question 10 is printed on the next page.

10 **(a)** Solve $\tan 3x = -1$ for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ radians, giving your answers in terms of π. [4]

(b) Use your answers to **part (a)** to sketch the graph of $y = 4\tan 3x + 4$ for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ radians on the axes below. Show the coordinates of the points where the curve meets the axes.



[3]